Supplemental Information for

Ultrafast optical phase-sensitive ultrasonic detection via dual-comb multiheterodyne interferometry

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1. I/Q demodulation

The phase information induced by the acoustic pressure is modulated to the down-converted IF (intermediate frequency) signal. In order to avoid the intensity effect, we use I/Q demodulation to extract the phase information based on digital zero IF (ZIF) architecture [46], shown in Fig. S1.

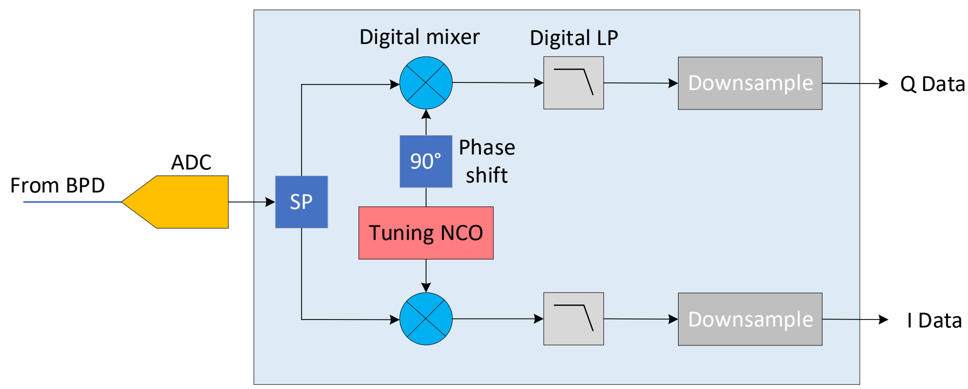


Fig. S1. Signal chain of I/Q demodulation based on ZIF architecture. ADC, analog-to-digital convertor; SP, digital power splitter; NCO, numerically controlled oscillator; LP, lowpass filter; Q, Quadrature; I, In phase.

The IF signal from the BPD in the DCMHI is sampled by the ADC. Two same signal chains after the digital power splitter are mixed with the digital LO signals, respectively. The digital LO signals are generated by a tuning numerically controlled oscillator (NCO) with a 90-degree phase shift at the same time. The frequency of the NCO is set equal to the IF signal. The digital lowpass filter is used to filter out the mirror signals after the digital mixer. After the downsample, the IF signal is down-converted to digital baseband and expressed as follow:

$$\begin{aligned} \text{Q data} &= A_S(t)\cdot A_{NCO}\sin\left[\varphi_S(t)-\varphi_{NCO}\right] \\ \text{I data} &= A_S(t)\cdot A_{NCO}\cos\left[\varphi_S(t)-\varphi_{NCO}\right]. \end{aligned} \tag{S1}$$

where $A_S(t)$ is the instantaneous amplitude of the IF signal, $\varphi_S(t)$ is the instantaneous phase of the IF signal, and A_{NCO}, φ_{NCO} is the amplitude and phase of NCO respectively, which can be precisely controlled. Thus, the phase of IF signal can be extracted as:

$$\varphi_S(t)=\text{actan}\left(\text{Q data / I data}\right). \tag{S2}$$

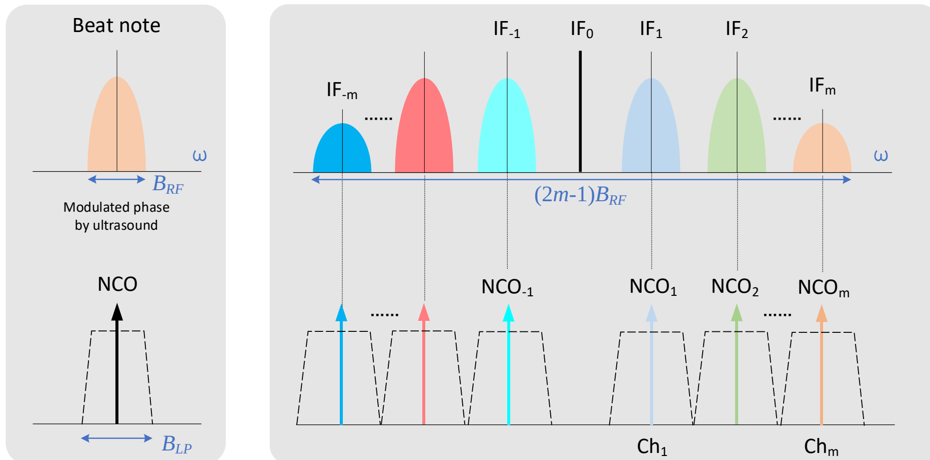
Due to the precise delay control in the digital domain, the amplitudes of the two I/Q signals are completely matched, so the influence of the amplitude is excluded during the phase extracting.

For phase detection for ultrasound, the flexibly settable phase value of NCO can improve the dynamic range. In addition to phase noise, the phase term φ_S(t) contains the initial phase of the beat frequency signal and the phase change induced by the ultrasound. Thanks to high coherence of dual-comb, the initial phase of each beating signal is the constant which can be compensated by adjusting the phase value φ_{NCO} of NCO. This results in detectable unambiguous phase change induced by ultrasound from negative π to positive π.

If the pressure induced by the ultrasound is too large along the working distance l, the phase change may be exceeded 2π, which will cause phase reversal in Eq. S2. In this case, we can synthesize two adjacent pairs of optical comb tones to execute the phase unwrap to expand sound pressure measurement range.

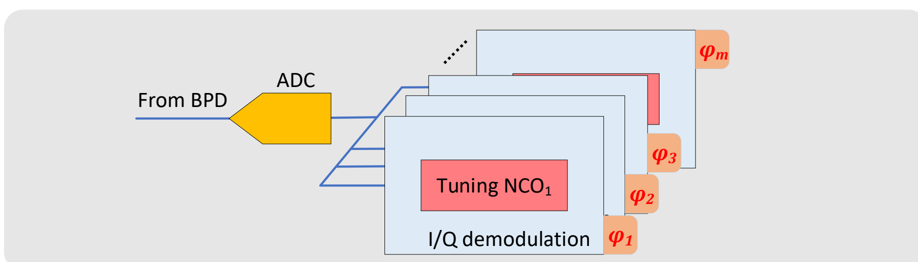
2. Digital channelized I/Q demodulation in the DCMHI

In the DCMHI, several beat notes will be generated in the BPD at the same time. In order to get the phase term of each beat note, we perform the channelized I/Q demodulation by constructing different NCO operated at different frequencies. The basic concept is described as follow:



(a) I/Q demodulation (b) Demodulated signal with multi IFs is mixed with different NCO

$$\mathrm{NCO}_m = \mathrm{IF}_m = \begin{cases} m(\omega_{S,r} - \omega_{LO,r}) + \omega_{AOFS} & m = 0,1,2,3\ldots \\ m(\omega_{S,r} - \omega_{LO,r}) - \omega_{AOFS} & m = -1,-2,-3\ldots \end{cases}$$



(c) Channelized I/Q demodulation

Fig. S2. The concept of channelized I/Q demodulation to extract multi phases at the same time. (1) Single IF signal mixed with NCO to perform I/Q demodulation. (b) Demodulated signal with multi-IFs is mixed with different NCO to down-convert to baseband. (c) Channelized I/Q demodulation.

From the experiment setup, the center frequency of dual-EOC is the same. Meanwhile, the AOFS is used to avoid beating note frequency aliasing and DC noise in the BPD. Thus, the frequency of each beating note is fixed and known. Ideally, the beat notes of the BPD have higher frequency IF signals,

$$\mathrm{IF}_{\text{high frequency}} = \begin{cases} m\omega_{S,r} - n\omega_{LO,r} + \omega_{AOFS} & m,n = 0,1,2,3\ldots \\ m\omega_{S,r} - n\omega_{LO,r} - \omega_{AOFS} & m,n = -1,-2,-3\ldots. \\ m \neq n \end{cases} \tag{S3}$$

However, in order to avoid these IF signals with high frequency from being sampled, the bandwidth of BPD used in the setup is 1.6 GHz (Thorlabs, PDB480C-AC), which is less than the repetition rate ($\omega_{S,r}/2\pi$ = 12 GHz) of each EOC. Thus, these IF signals cannot be detected in the BPD.

After sampling by high-speed ADC, the beating note from the BPD is separated into different channels to mix with different digital LO generated by NCO. Each frequency of NCO is equal to the IF and shown as:

$$\mathrm{NCO}_m = \mathrm{IF}_m = \begin{cases} m(\omega_{S,r} - \omega_{LO,r}) + \omega_{AOFS} & m = 0,1,2,3... \\ m(\omega_{S,r} - \omega_{LO,r}) - \omega_{AOFS} & m = -1,-2,-3... \end{cases} . \tag{S4}$$

where ω_{AOFS} is the angular frequency of AOFS. The bandwidth of digital lowpass filters in different channels is the same and numerically controlled, which is usually set to the desired acoustic bandwidth. Finally, the phase $\varphi_m(t)$ and $\varphi_{ref,m}(t)$ of these beat notes in signal and reference BPD are extracted at the same time. The advantages of channelized I/Q demodulation are twofold:

-First, the receiving spur free dynamic range of the system is improved. The total noise bandwidth mB_{RF} is reduced by a factor of m to B_{RF}.

-Second, the speed of data processing is improved. The channelized I/Q demodulation can be processed in the parallel in field programmable gate array. Then combined with the high frame rate of dual-comb, transient phase sensing and imaging can be realized.

3. EOC generation and microwave synchronization

The central mode of each EOC is formed from a 1555 nm CW fiber laser (NKT photonics, Koheras BASIK MIKRO E15), which has ultra-low phase noise and hertz linewidth without stabilizing to an ultra-low optical reference cavity. The output of the fiber laser is fed into an intensity modulator (EOSPACE, OC-768) driven by the microwave signal to generate flat top optical pulse train. A single lithium-niobate phase modulator (EOSPACE, 20 GHz@3 dB bandwidth) with low half-wave voltage ($V\pi$=2.9 V@1 GHz) driven by the same microwave signal is then added to generate a periodically chirped waveform by the time-to-frequency mapping due to strong quadratic phase modulation [42,47]. The total number of comb tones generated by a single phase modulator is determined by the half-wave voltage of the modulator and microwave drive power. Due to the limitation of the maximum acceptable microwave power, several modulators can be cascaded to increase the number of comb tones. In actual measurements, we found that too many comb teeth are not needed for accumulation. To simplify the system, one phase modulator is only used in the setup. In this case, each EOC can generate approximately 20 comb tones (-3 dB power flatness) with the drive power of 29 dBm. The reason for generating flat comb tones is to ensure the dynamic range of the detected signals and optimal sampling for all beat signals in the ADC.

Two DROs (Microwave Dynamics, DRO-1000) with center frequencies at 12 GHz and 12.2 GHz are used to drive two EOCs. Both of them are synchronized to a rubidium clock with 10 MHz output by the phase-locked loop (Analog Device ADF4159). After phase locking, two driver amplifiers are used to amplify the output signal to 30 dBm. Before feeding into different modulators, a microwave bandpass filter with a 10 MHz passband@3 dB and 200 MHz stopband@60 dB at the corresponding center frequency is added to suppress out-of-band phase noise and harmonic spurs. The insertion loss of the microwave filter is about 1 dB.

4. Shot noise limitation in the BPD

There are several noise mechanisms to affect the sensitivity of the photodetector. For p-i-n photodiodes usually used in most BPD, the main influences to be considered are [48]:

- ✧ Quantum efficiency: Since the photons absorbed by the diode do not all generate electron-hole pairs, the quantum efficiency of the photodiodes cannot reach 1. The photodiode responsivity R is used to describe.
- ✧ Shot noise: It is caused by the random fluctuations of the current carriers generated by electron-hole pairs in PD. For a suitable value of detection power P_S, the shot noise $n(t)$ follows a normal distribution with mean zero, as shown:

$$n(t) = 2qRP_S B_{RF}. \tag{S5}$$

  Where q is the elementary charge (1.602×10^{-19} As), and B_{RF} is the bandwidth of the PD.
- ✧ Thermal Noise: The thermal noise N_{th} is produced by the thermal motion of electrons. It is especially evident in transimpedance amplification in the BPD.

Considering the influence of the above factors, the signal-to-noise ratio of coherent reception is:

$$SNR = \frac{4R^2 \left\|E_s^*(t)\right\| \cdot \left\|E_{LO}(t)\right\|}{B_{RF}(2qR\left\|E_{LO}(t)\right\| + N_{th})}. \tag{S6}$$

When the shot noise power exceeds the thermal noise power by far, the SNR expression will be changed to:

$$SNR = \frac{2R\left\|E_s^*(t)\right\|}{qB_{RF}}. \tag{S7}$$

The condition is called shot noise limitation, in which the receiver noise is mainly dominated by shot noise.

From the Eq. S5 and S6, the shot noise power density will be increased at the same rate with increasing LO power. Thus, it is necessary to define an appropriate LO power to make the BPD work at the shot noise limitation condition. In order to compare the different noises, the noise must be equivalent to the input. The thermal noise of a transimpedance amplifier in current source form is expressed as [49]:

$$N_{th} = \overline{I^2_{n,Rd}} = \frac{4kT}{R_d} B_{RF}. \tag{S8}$$

where k is Boltzmann's constant (1.38×10^{-23} J/K), T is Kelvins temperature, and R_d is equivalent transimpedance. Demanding that the shot noise power should exceed the thermal noise power by at least a factor of ten, thus

$$\frac{n(t)}{N_{th}} = 10 \Rightarrow \left\|E_{LO}(t)\right\| = \frac{20kT}{qRR_d}. \tag{S9}$$

From the technical data of BPD (Thorlabs, PDB480C-AC), R_d = 50Ω, R = 0.95 A/W, other T = 298.15 K (room temperature). We can get the required LO power is 1.08 mW (around 0.33 dBm). So that the power of the LO EOC fed into the BPD is set to 1 dBm to ensure the system working at shot noise limitation condition.

5. Phase noise performance of the system

For electro-optic frequency combs, the phase noises on the n^{th} comb line of the signal and LO comb are given by

$$\begin{aligned} \varphi_{sig}^{n}(t) &= \varphi_c(t) + n\varphi_{RF-sig}(t), \\ \varphi_{LO}^{n}(t) &= \varphi_c(t) + n\varphi_{RF-LO}(t) . \end{aligned} \tag{S10}$$

where $\varphi_c(t)$ is the phase noise of seed laser, $\varphi_{RF\text{-}sig}(t)$, $\varphi_{RF\text{-}LO}(t)$ are the phase noise of the driving signals for dual-comb respectively. The laser phase noise $\varphi_c(t)$ will be canceled at the coherent receiver assuming that the optical path length difference between the two combs is significantly shorter than the coherence length of the seed laser. That can be achieved by using a narrow-linewidth laser. Thus, the phase term $\Delta\varphi_{S\text{-}LO}$ in Eq. 6 equals to：

$$\Delta\varphi_{S-LO}(t) = n\varphi_{RF-sig}(t) - n\varphi_{RF-LO}(t) . \tag{S11}$$

when the driving signals for dual-comb are referenced to a common reference (Rb clock), the phase noises are not completely independent. The phase noise of this common reference will be scaled up within the PLL and contributes to the overall phase noise of the PLL output signal. At low offset frequencies, the reference phase noise (typically, $1/f$ noise) will dominate the overall phase noise of the PLL. Within the loop bandwidth, the phase-frequency detector (PFD) will tend to contribute the majority of phase noise. The DRO noise dominates outside the loop bandwidth.

For $\varphi_N(t)$, the most direct source of noise comes from link jitter, shown as:

$$\varphi_N(t) = \omega_S\tau_S - \omega_{LO}\tau_{LO} . \tag{S12}$$

In this letter, the phase term $\varphi_N(t)$ can be effectively eliminated by auxiliary optical path.

6. Frequency response measurement

The frequency response of the system is measured by inputting a normalized or calibrated broadband signal, which in turn measures the amplitude and frequency response of the system output signal. Typically, the broadband signal can be a continuous broad-spectrum signal (such as a swept signal), or multiple monophonic signals. Due to experimental constraints, we use multiple single-tone signal for measurements, which is generated by different ultrasound transducers.

We also used hydrophone and DCMHI to measure the ultrasonic signal under the same driving signal respectively, and obtained the sensitivity at different frequencies. Since the system responds to low frequencies in much the same way as the hydrophone, as shown in the inset of Fig. 6. The sensitivities of other frequencies are normalized to the sensitivity at 1 MHz, and the results in Fig. 6 are plotted.